Fuzzy Sequencing Problem in Enneadecagonal Fuzzy Number

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Abstract

We intend a new fuzzy number named as Enneadecagonal fuzzy number and its membership function in this paper. We set out a way of dealing with to solve fuzzy sequencing problem where processing time taken as Enneadecagonal fuzzy numbers. Fuzzy sequencing problem are changed into a crisp valued sequence problem which is illustrated with a numerical example.

Keywords: Enneadecagonal Fuzzy number, Membership function, Fuzzy sequencing problem.

1. Introduction:
The fuzzy set is brought into use by Zadeh [1]. In our daily life, we meet many decision making situations. Each situation is remarkable. We take decisions on each situation is either clear or not clear. Sequencing which gives an initiative of the order in which things are happening or approaching in occurrence. There we are in the position in decision making that is job sequencing, game theory and etc. Job sequencing problem is most important problem in the computer field. Job sequencing problem is one of the most and significant application of Optimization techniques. The major role of the sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the total amount of the time required to complete the process of all the jobs. Raju and Jayagopal [2] introduced the Icosagonal fuzzy number. Both of them have developed and introduced Icosikaioctagonal fuzzy number [3]. In this paper, we have explored the sequencing problem using Enneadecagonal fuzzy number. We have used Enneadecagonal fuzzy number and its membership function. Using Pascal’s triangular Graded Mean fuzzy sequencing problem can be transformed into a crisp valued problem and is explained by appropriate examples. The processing time is considered to be Enneadecagonal fuzzy number. By finding an answer to crisp sequencing problem, we obtain the optimal order, idle time and total elapsed time for each machine.

2. PRELIMINARIES
In this section, we give the preliminaries that are required for this study.

Definition 2.1. A fuzzy set A is defined by $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. Here x is crisp set A and $\mu_A(x)$ is membership function in the interval [0,1].

Definition 2.2. The fuzzy number A is a fuzzy set whose membership function must satisfy the following conditions.

(i) A fuzzy set A of the universe of discourse X is convex
(ii) A fuzzy set A of the universe of discourse X is a normal fuzzy set if $x_i \in X$ exists
(iii) $\mu_A(x)$ is piecewise continuous

2.3 Ranking of Enneadecagonal fuzzy number:
Let I be a normal Enneadecagonal fuzzy number. The value $M(I)$, called as measure of I is calculated as
\[ M(I) = \frac{e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8 + e_9 + e_{10} + e_{11} + e_{12} + e_{13} + e_{14} + e_{15} + e_{16} + e_{17} + e_{18} + e_{19}}{19} \]

where \( 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq 1 \)

3. Definition [3] A fuzzy number \( A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, \ldots, a_9) \) is Enneadecagonal fuzzy number and its membership function is given by

\[
\mu_E(x) = \begin{cases} 
0, & \text{for } x < e_1 \\
k_1 \left( \frac{x - e_1}{e_2 - e_1} \right), & \text{for } e_1 \leq x \leq e_2 \\
k_1, & \text{for } e_2 \leq x \leq e_3 \\
k_1 + (k_2 - k_1) \left( \frac{x - e_3}{e_4 - e_3} \right), & \text{for } e_3 \leq x \leq e_4 \\
k_2, & \text{for } e_4 \leq x \leq e_5 \\
k_2 + (k_3 - k_2) \left( \frac{x - e_5}{e_6 - e_5} \right), & \text{for } e_5 \leq x \leq e_6 \\
k_3, & \text{for } e_6 \leq x \leq e_7 \\
k_3 + (k_4 - k_3) \left( \frac{x - e_7}{e_8 - e_7} \right), & \text{for } e_7 \leq x \leq e_8 \\
k_4, & \text{for } e_8 \leq x \leq e_9 \\
k_4 + (1 - k_4) \left( \frac{x - e_9}{e_{10} - e_9} \right), & \text{for } e_9 \leq x \leq e_{10} \\
k_4 + (1 - k_4) \left( \frac{e_{10} - x}{e_{11} - e_{10}} \right), & \text{for } e_{10} \leq x \leq e_{11} \\
k_4, & \text{for } e_{11} \leq x \leq e_{12} \\
k_3 + (k_4 - k_3) \left( \frac{e_{12} - x}{e_{13} - e_{12}} \right), & \text{for } e_{12} \leq x \leq e_{13} \\
k_3, & \text{for } e_{13} \leq x \leq e_{14} \\
k_2 + (k_3 - k_2) \left( \frac{e_{14} - x}{e_{15} - e_{14}} \right), & \text{for } e_{14} \leq x \leq e_{15} \\
k_2, & \text{for } e_{15} \leq x \leq e_{16} \\
k_1 + (k_2 - k_1) \left( \frac{e_{16} - x}{e_{17} - e_{16}} \right), & \text{for } e_{16} \leq x \leq e_{17} \\
k_1, & \text{for } e_{17} \leq x \leq e_{18} \\
k_1 \left( \frac{e_{18} - x}{e_{19} - e_{18}} \right), & \text{for } e_{18} \leq x \leq e_{19} \\
0, & \text{for } x > e_{19}
\end{cases}
\]
4. Processing of ‘n’ jobs through ‘2’ machines:
   Let ‘n’ jobs $A_1, A_2, \ldots, A_n$ be processing through 2 machines that is $M_1, M_2$ respectively. Let $R_{ij}$ be the fuzzy processing time taken by $i^{th}$ job to be done by $j^{th}$ machine. Using Johnson method, we can find optimal sequence, total elapsed time and idle time on machines. Here fuzzy times are taken as Enneadecagonal fuzzy number.

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine M I</th>
<th>Machine M II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$R_{11}$</td>
<td>$R_{12}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$R_{21}$</td>
<td>$R_{22}$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$R_{31}$</td>
<td>$R_{32}$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>$R_{41}$</td>
<td>$R_{42}$</td>
</tr>
</tbody>
</table>

5. Pascals Triangular graded mean for Enneadecagonal Fuzzy Number:
   Consider $G_{\cos kect} = \left(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, \ldots, a_{19}\right)$ be an Enneadecagonal fuzzy number. By taking the co-efficient of fuzzy numbers from Pascal’s triangle. Then the formula of Pascal’s triangular graded mean for Enneadecagonal fuzzy number is

   $$G(A) = \frac{1a_1 + 18a_2 + 153a_3 + 816a_4 + 3060a_5 + 8568a_6 + 18564a_7 + 31824a_8 + 43758a_9 + 48620a_{10} + 43758a_{11} + 31824a_{12} + 18564a_{13} + 8568a_{14} + 3060a_{15} + 816a_{16} + 153a_{17} + 18a_{18} + 1a_{19}}{262144}$$

   The coefficients of $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, \ldots, a_{19})$ are 1, 18, 153,816,3060,8568,18564,31824,43758,48620,43758,31824,18564,8568,3060,816,153,18,1 respectively. These coefficients are taken from the Pascal’s triangles.

   **Step 1:** Using Pascal graded mean, fuzzy sequencing problem is converted to a crisp valued problem.
   **Step 2:** The optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.
   **Step 3:** After finding the optimal sequence. Determine the total elapsed fuzzy time and also the fuzzy ideal time on machines.

7. Numerical example:
   We are taking into account the fuzzy sequence problem. Let us take the processing time of 4 jobs are given in which all the elements are fuzzy quantifiers which signalize the linguistic variables that are taking the place of Enneadecagonal fuzzy numbers.
These qualitative data are transformed into quantitative data and which is shown in the below table. The processing time is between 1 to 76 and the minimum value is considered as 1 and maximum value is considered as 76 and is shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Machine I</th>
<th>Machine II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jobs</td>
<td>1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19</td>
<td>39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57</td>
</tr>
<tr>
<td>A1</td>
<td>12345678910111213141516171819</td>
<td>39404142434445464748495051525354555657</td>
</tr>
<tr>
<td>A2</td>
<td>20212223242526272829303132333435363738</td>
<td>58596061626364656667686970717273747576</td>
</tr>
<tr>
<td>A3</td>
<td>58596061626364656667686970717273747576</td>
<td>20212223242526272829303132333435363738</td>
</tr>
<tr>
<td>A4</td>
<td>39404142434445464748495051525354555657</td>
<td>12345678910111213141516171819</td>
</tr>
</tbody>
</table>

The problem is shown in the table.

Apply Pascal’s triangular graded mean for Enneadecagonal fuzzy number, the fuzzy valued time connected to respective valued time.

<table>
<thead>
<tr>
<th></th>
<th>R11=10</th>
<th>R12=48</th>
</tr>
</thead>
<tbody>
<tr>
<td>R21=29</td>
<td>R22=67</td>
<td></td>
</tr>
<tr>
<td>R31=67</td>
<td>R32=29</td>
<td></td>
</tr>
<tr>
<td>R41=48</td>
<td>R42=10</td>
<td></td>
</tr>
</tbody>
</table>

The Optimum sequence is $R_{11} \ R_{31} \ R_{41} \ R_{31}$

Total elapsed time and idle time

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine I</th>
<th>Machine II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time in</td>
<td>Time out</td>
</tr>
<tr>
<td>A1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>A2</td>
<td>10</td>
<td>39</td>
</tr>
<tr>
<td>A3</td>
<td>39</td>
<td>87</td>
</tr>
<tr>
<td>A4</td>
<td>87</td>
<td>154</td>
</tr>
</tbody>
</table>

Total Elapsed time = 183 Hrs
Idle time on Machine I = 29 Hrs
Idle time on Machine II = 29 Hrs
8. Conclusion:
In this paper, we have solved fuzzy sequencing problem by using Enneadecagonal fuzzy numbers. The Fuzzy sequencing problem of Enneadecagonal Fuzzy Numbers has been transformed into crisp sequencing problem using Pascal’s Graded mean formula. By this method, we obtained the optimal sequence, total elapsed time and idle time for each machine.

References:

